The Graviton as a Bound State and the Cosmological Constant Problem

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Abstract

The graviton is pictured as a bound state of a fermion and anti-fermion with the spacetime metric assumed to be a composite object of spinor fields, based on a globally Lorentz invariant action proposed by Hebecker and Wetterich. The additional degrees of freedom beyond those of the graviton are described by Goldstone boson gravitational degrees of freedom. If we assume that the fermion is a light neutrino with mass $m_\nu \sim 10^{-3}$ eV, then we obtain the effective vacuum density $\bar{\rho}_\lambda \sim (10^{-3}\, eV)^4$, which agrees with the estimates for the cosmological constant from WMAP and SNIa data.

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1 Introduction

The cosmological constant problem is generally accepted to be one of the most serious paradoxes in modern particle physics and cosmology [1, 2]. Let us express the cosmological constant in the form

$$\lambda = \lambda_0 + 8\pi G\rho_{\rm vac}, \tag{1}$$

where λ_0 is the bare cosmological constant in the Einstein-Hilbert action, and $\rho_{\rm vac}$ is the vacuum contribution from the vacuum expectation value

$$\langle T_{\mu\nu}\rangle = g_{\mu\nu}\rho_{\rm vac} + \text{higher curvature terms}. \tag{2}$$

We know from cosmological observations that

$$\rho_\lambda \sim \rho_{\rm crit} = \frac{3H_0^2}{8\pi G} = 8\times 10^{-47}\, h_0^2\, GeV^4, \tag{3}$$

where $\rho_\lambda = \lambda/8\pi G$ and $h_0 = H_0/(100\,{\rm km}\,s^{-1}\,{\rm Mpc}^{-1}) \sim 0.6$.

From standard field theory calculations, we find that calculations of ρ_λ differ from the value $\rho_\lambda \sim 10^{-47}\, GeV^4 \sim (10^{-3}\, eV)^4$, obtained from WMAP and SNIa data [3, 4, 5, 6], by 10^{55} at the electroweak energy scale$\sim$ 250 Gev, and by 10^{121} in natural reduced Planck energy units.

The significant discrepancy between the expectations of particle physics and the cosmological data strongly suggests that we cannot picture the graviton as an "ordinary" particle, such as the photon and the W and Z mesons of the standard model [7]. The shift in the action $S\rightarrow S-S_{\rm vac}$ is made nontrivial by the fact that the graviton couples to all forms of matter universally, whereas the photon only sees electric charge and the gluon only sees color charge. Thus, the solution of the cosmological constant problem must reside in a radical change in how we picture the graviton. Various possible explanations have been proposed for how this change should be implemented [8, 9]. The cosmological constant problem has been exacerbated by the problem of dark energy. If the vacuum energy density is the source of dark energy, as is suggested by the WMAP data analysis, then we no longer seek to find an explanation for λ being zero, but we must explain the incredible smallness of λ when fitted to the cosmological data.

In the following, we shall investigate the consequences of picturing the graviton as a bound state condensate of a fermion and an anti-fermion. It has been known for a long time that by postulating something akin to Sakharov's induced gravity [10], generically a one loop action produces an effective action containing the Einstein-Hilbert action with a cosmological constant term [11]. We will base our desciption of the graviton condensate on a recently proposed model of composite gravitons in which the metric tensor of spacetime is described by the expectation value of a vierbein spinor bilinear form [12, 13]. By identifying the fermion field as a light neutrino with mass $m_\nu \sim 10^{-3}$ eV, we predict that the effective vacuum density $\bar\rho_\lambda \sim (10^{-3}\, eV)^4$ in agreement with estimates from the WMAP and SNIa data [3, 4, 5, 6].

2 Bound State Model

Let us assume that the graviton is a bound state of a fermion particle. The metric is given by

$$g_{\mu\nu} = \langle E^a_\mu E^b_\nu \eta_{ab} \rangle, \tag{4}$$

where $M^*_{PL} = M_{PL}/\sqrt{8\pi G} = 2.4 \times 10^{18}$ GeV is the reduced Planck mass and [12, 13]:

$$E^a_\mu = \frac{i}{2}\biggl(\frac{1}{M^*_{PL}}\biggr)\bar{\psi}\gamma^a\partial_\mu\psi + h.c. \tag{5}$$

We assume that $E = \det(E^a_\mu) \neq 0$, so that E^a_μ and $g^{\mu\nu}$ are well defined. We have scaled ψ so that it, E^a_μ and $g_{\mu\nu}$ have mass dimension 0.

A diffeomorphism and *globally* Lorentz invariant action is [12, 13]

$$S = M^{*4}_{PL} \int d^4x E(x), \tag{6}$$

where

$$E = \frac{1}{4!}\epsilon^{\mu_1 \ldots \mu_4}\epsilon_{a_1 \ldots a_4} E^{a_1}_{\mu_1} \ldots E^{a_4}_{\mu_4} = \biggl(\frac{1}{M^{*4}_{PL}}\biggr)\mathcal{O}, \tag{7}$$

and $\mathcal{O}$ is a bilinear spinor operator.

In contrast to Einstein gravity, S is not locally Lorentz invariant, because the covariant derivative D_μ contains the Christoffel connection $\Gamma^\lambda{}_{\mu\nu}$ but not a spin connection. This leads to a generalized gravity theory in which the vierbein contains additional degrees of freedom that are not described by $g_{\mu\nu}$. These additional degrees of freedom produce new invariants not present in Einstein gravity, which are globally Lorentz invariant but not locally Lorentz invariant. A nonlinear field decomposition $E^a_\mu = e^b_\mu H^a_b$ can be employed, where e^b_μ describes the standard Einstein gravity vierbein and H^a_b describes the new degrees of freedom, associated with Goldstone boson excitations due to the spontaneous breaking of a global symmetry. The H^a_b would correspond in Einstein gravity to gauge degrees of freedom of the local Lorentz transformation group and therefore not be present in an invariant action. In the Hebecker-Wetterich (HW) action there exist new propagating massless particles associated with the new degrees of freedom.

An estimate of the one loop order of fermionic fluctuations, $\Gamma_{[E]}$, is given by [12, 13]:

$$\Gamma_{[E]} = \alpha \int d^4x \langle E \rangle - \frac{1}{2}\ln(\langle E \rangle \mathcal{D}), \tag{8}$$

where $\alpha = (-1)^5 M^{*4}_{PL}$ and $\langle E\rangle\mathcal{D}$ is the second functional derivative of the bosonic action S_B with respect to ψ. The bosonic action S_B is defined in terms of the partition function Z as a functional integral over the fermion field ψ and a boson field χ. Moreover, $\mathcal{D} = \langle E^\mu_a\rangle\gamma^a\hat{D}_\mu$ where $\hat{D}_\mu = \partial_\mu + (1/2\langle E\rangle)\langle E^a_\mu\rangle\partial_\nu(\langle EE^\nu_a\rangle)$.

The quantum field equations that follow from a variation of the effective action $\Gamma(E)$ are given by

$$\frac{\delta\Gamma(E)}{\delta\langle E^a_\mu\rangle} = J^\mu_a, \tag{9}$$

where $J^\mu_a = 0$ in empty spacetime. An energy-momentum tensor which includes all forms of matter and radiation is given by

$$T^{\mu\nu} = \langle E^{-1}E^{a\mu}\rangle J^\nu_a. \tag{10}$$

The effective one loop action, obtained from the HW action (6), containing two derivatives, which is invariant under global Lorentz transformations and diffeomorphism transformations is

$$\Gamma_{(2)} = \frac{M^{*2}_{PL}}{2}\int d^4x\langle E\rangle[-R + \tau(D^\mu\langle E^\nu_a\rangle D_\mu\langle E^a_\nu\rangle - 2D^\mu\langle E^\nu_a\rangle D_\nu\langle E^a_\mu\rangle)$$

$$+\beta D_\mu\langle E^\mu_a\rangle D^\nu\langle E^a_\nu\rangle] + M^{*2}_{PL}\int d^4x\langle E\rangle\lambda, \tag{11}$$

where

$$D_\mu\langle E^a_\nu\rangle = \partial_\mu\langle E^a_\nu\rangle - \Gamma^\lambda{}_{\mu\nu}\langle E^a_\lambda\rangle. \tag{12}$$

Here, λ is the cosmological constant in the effective action, $\lambda = \rho_\lambda/M^{*2}_{PL}$, $\Gamma^\lambda{}_{\mu\nu}$ and R are the Christoffel symbols and Ricci scalar obtained from the metric $g_{\mu\nu}$, and indices are raised and lowered with the metric tensor. Due to the missing spin connection in D_μ, the two terms multiplying τ and β are invariant only under global Lorentz transformations.

3 Resolution of the Cosmological Constant Problem

According to Eq.(7), the cosmological constant term $\sqrt{-g}\rho_\lambda$ in the Einstein-Hilbert action is replaced in the effective action (11) by

$$\bar{\rho}_\lambda = \langle E\rangle\rho_\lambda = \left(\frac{1}{M^{*4}_{PL}}\right)\langle\mathcal{O}\rangle\rho_\lambda. \tag{13}$$

We set the mass scale of $\langle \mathcal{O} \rangle$ by

$$\langle \mathcal{O} \rangle \sim O(m^4), \tag{14}$$

where m is a low energy mass.

We now have

$$\bar{\rho}_\lambda \sim \left(\frac{m}{M^*_{PL}}\right)^4 M^{*2}_{PL}\lambda. \tag{15}$$

We choose $\lambda \sim M^{*2}_{PL}$ which is the result obtained for a cutoff $\Lambda_c \sim M^{*2}_{PL}$ in natural reduced Planck units and we obtain

$$\bar{\rho}_\lambda \sim m^4. \tag{16}$$

Identifying the fermion field ψ with the lightest neutrino field ψ_ν with a mass $m = m_\nu \sim 10^{-3}$ eV [14, 15, 16], we get

$$\bar{\rho}_\lambda \sim (10^{-3}\, eV)^4, \tag{17}$$

which agrees with estimates for the vacuum density obtained from the WMAP and SNIa data [3, 4, 5, 6].

The terms multiplying τ and β in (11) must be small in order to avoid deviations from Einstein gravity and observations. The contribution associated with β vanishes in one loop order. An analysis [12, 13] of these contributions shows that for $\beta = 0$, the terms multiplying τ do not affect the lowest order post-Newtonian gravity, and the Schwarzschild solution. For the Friedmann-Robertson-Walker cosmological solution, a value of the Planck mass is obtained which differs from Einstein gravity, which can be checked by comparing it with a local gravitational measurement of Newton's constant G.

The $\rho_{\rm vac}$ obtained from $T^{\mu\nu}_{\rm vac}$ in (10) contains all contributions, including those arising from phase transitions, such as chiral symmetry breaking, QCD gluon condensates and Higgs spontaneous symmetry breaking in the standard model.

A determination of the absolute values of neutrino masses is difficult to achieve experimentally. Limits on neutrino masses can be obtained from neutrino oscillation models, tritium decay and cosmological bounds [15, 16]. From the mass hierarchy of three-neutrino mixing models one finds in a normal scheme the effective mass of the lightest neutrino has a value between about 3×10^{-3} eV and 2×10^{-2} eV. Hopefully, future experiments will narrow down the range of values of the lightest neutrino mass.

4 Conclusions

By using a generalized gravity theory based on a bilinear operator form developed by Hebecker and Wetterich [12, 13], we have predicted a vacuum density $\bar{\rho}_\lambda \sim (10^{-3}\, eV)^4$ in agreement with λCDM model estimates from WMAP and SNIa data, when we identify the bound state fermion associated with the graviton condensate with a light neutrino with mass $m = m_\nu \sim 10^{-3}$ eV.

By identifying ψ with a light neutrino field, we have predicted the correct magnitude of $\bar{\rho}_\lambda$ that fits the λCDM model interpretation of dark energy. This suggests that we describe the dark energy as graviton condensates formed from fluctuating light neutrinos. The source of dark energy would be light neutrino and anti-neutrino condensates.

It would be interesting to investigate further whether there exists a self-consistent gravity theory for composite gravitons, based on fermion bilinear correlation functions that exhibits local Lorentz invariance as opposed to the global Lorentz invariance of the HM model. Up till now no such model has been discovered.

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